

The article discusses several approaches [1-3] to a description of creep under complex loading, making it possible to take account of the anisotropic character of the hardening of a material with not fully developed creep. An attempt is made to take account of the different behavior of a material with increasing and decreasing stresses. The results of predictions made using various theories are compared with experimental data from investigations of the creep of AK4-1 alloy under the combined action of elongation and torsion, under conditions of nonproportional stepwise loading. Comparison with experiment shows that, under complex loading, a theory based on the hypothesis of isotropic hardening does not describe the experimental data satisfactorily. Considerably better results are given by theories which take account of the anisotropic character of the hardening of materials with not fully established creep.

1. The experiments were carried out on tubular samples at a temperature of 175°C and a common duration of 100 h. The method used in the tests is described in [4], which gives the results of investigation of the creep of AK4-1 alloy with a complex loading state, under constant and proportionally varying loads.

The program of tests shown in Table 1 was carried out under conditions of nonproportional loading.

In the first experiment the loading was effected in two stages, with a rising intensity of the stresses. In the second and third experiments the form of the state of stress was varied stepwise, from monoaxial elongation (torsion) in the first stage to pure torsion (elongation) in the second stage. In the fourth and fifth experiments the change in the form of the state of stress was carried out consecutively in four stages, from monoaxial elongation (torsion) in the first stage to pure torsion (elongation) in the fourth stage. In the last four experiments the intensity of the stresses remained constant during all stages of the loading. From two to four samples were tested in each experiment.

In Figs. 1, 2, and 3 the averaged results of the tests are illustrated by the lines with small circles, in the form of creep curves. The numbers in the upper right-hand corner denote the number of the experiments; γ is the shear deformation of the creep; ϵ is the axial deformation of the creep.

It was shown in [4] that, under constant and proportionally varying loads, relationships based on the hypothesis of isotropic hardening are valid:

$$p'_{kj} = \frac{3}{2} p'_i / \sigma_i^{-1} \sigma_{kj}^* \tag{1.1}$$

$$p'_i p_i^\alpha = k \sigma_i^n \tag{1.2}$$

$$p_i' = \left(\frac{2}{3} p'_{kj} p'_{kj} \right)^{1/2}, \quad p_i = \int_0^t p_i' dt, \quad \sigma_i = \left(\frac{3}{2} \sigma_{kj}^* \sigma_{kj}^* \right)^{1/2}$$

Here p'_{kj} are the components of the tensor of the deformation rates of the creep; σ_{kj}^* are the components of the deviator of the stress tensor; p_i' is the intensity of the deformation rates of the creep; p_i is a parameter of the hardening; σ_i is the intensity of the stresses; α, n, k are constants of the creep, for which the following values were obtained in [4]:

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TABLE 1

Number of experiment	Number of sample	σ	τ	σ_i	t	σ	τ	σ_i	t
		first stage				second stage			
1	9, 10	6	7.94	15	50	10.5	9.14	19	50
2	22, 25, 29, 43	15	0	15	50	0	8.67	15	50
3	24, 30, 31, 42	0	10.4	18	50	18	0	18	50
4	19, 32, 33, 41	15	0	15	25	10	6.45	15	25
5	16, 36, 51, 53	0	10.4	18	25	9	9	18	25
		third stage				fourth stage			
4	19, 32, 33, 41	6	7.94	15	25	0	8.67	15	25
5	16, 36, 51, 53	13	7.18	18	25	18	0	18	25

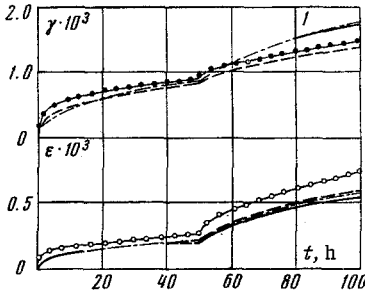


Fig. 1

$$\alpha = 1.5, \quad n = 7.5$$

$$k = 8.375 \cdot 10^{-20} \text{ (mm}^2\text{n/kg}^n \cdot \text{h)}$$

A check of Eqs. (1.1) and (1.2) under nonproportional loading showed considerable deviations of the calculated values of the components of the deformations from the experimental data (Figs. 1 and 2). [In Figs. 1, 2, 3, the solid lines, b, are plots of calculated curves obtained by integration of relationships (1.1) and (1.2).]

2. To describe creep under the conditions of a complex state of stress, article [1] postulates the existence of a potential function of the deformation rates of the creep, into which a mixed invariant of the components of the stress and deformation tensors is introduced as the parameter of the hardening q .

In accordance with the form of dependence (1.2), the potential function is constructed in the form

$$\Phi = k_0 (S^{1/2} q^{-1})^\alpha S^{(n+1)/2}$$

$$S = 3\sigma_{kj}^* \sigma_{kj}^*, \quad \sigma_{kj}^* = \sigma_{kj} - 1/3 \delta_{kj} \sigma_{ll}$$
(2.1)

Here δ_{kj} is a Kronecker symbol; $q = \sigma_{kj} p_{kj}$ for values of $\sigma_{kj} p_{kj} \geq 0$, and $q = 0$ for negative values of $\sigma_{kj} p_{kj}$; k_0 is a constant, which is expressed in terms of the corresponding constants from (1.2).

In the case of a monoaxial state of stress, the expression for the deformation rates of the creep $p_{kj}' = \partial \Phi / \partial \sigma_{kj}$ is transformed to the equation $p' p^\alpha = k \sigma^n$, where σ is the normal stress.

To calculate the components of the deformations of the creep with the combined action of elongation and torsion, the following dependences have been obtained:

$$\varepsilon_m = \varepsilon_{m-1} + \Delta \varepsilon_m$$

$$\gamma_m = \gamma_{m-1} + \Delta \gamma_m$$
(2.2)

$$\Delta \varepsilon_m = \frac{k (\sigma_m^2 + 3\tau_m^2)^{(n+\alpha-1)/2}}{(n+1) (\sigma_m \varepsilon_{m-1} + \tau_m \gamma_{m-1})^\alpha} \left[(\alpha + n + 1) \sigma_m - \frac{\alpha \varepsilon_{m-1} (\sigma_m^2 + 3\tau_m^2)}{\sigma_m \varepsilon_{m-1} + \tau_m \gamma_{m-1}} \right] \Delta t_m$$
(2.3)

$$\Delta \gamma_m = \frac{k (\sigma_m^2 + 3\tau_m^2)^{(n+\alpha-1)/2}}{(n+1) (\sigma_m \varepsilon_{m-1} + \tau_m \gamma_{m-1})^\alpha} \left[(\alpha + n + 1) 3\tau_m - \frac{\alpha \gamma_{m-1} (\sigma_m^2 + 3\tau_m^2)}{\sigma_m \varepsilon_{m-1} + \tau_m \gamma_{m-1}} \right] \Delta t_m$$
(2.4)

where τ is the tangential stress.

With a change in the stress-deformation state, when the value of q becomes negative or equal to zero, from (2.3) and (2.4) we obtain

$$\Delta \varepsilon_m / \Delta \gamma_m = - \varepsilon_{m-1} / - \gamma_{m-1}$$

Under these circumstances, the creep process will proceed in a direction opposed to the direction of the originally accumulated deformation before the reversion of the latter to zero.

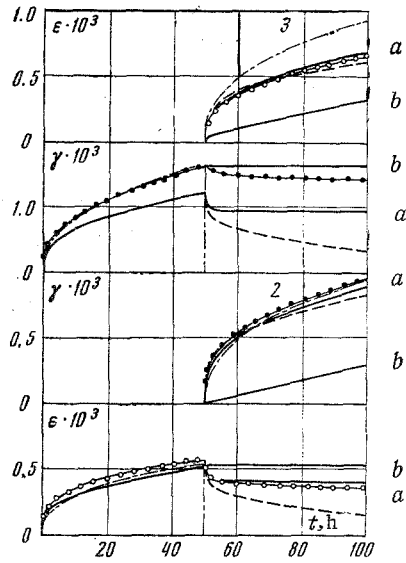


Fig. 2

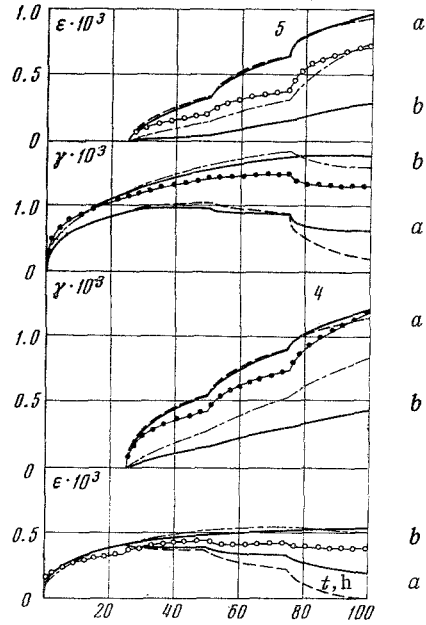


Fig. 3

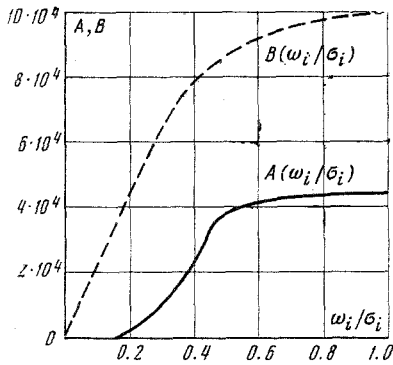


Fig. 4

The results of calculations using dependences (2.2)–(2.4) are shown in Figs. 1, 2, and 3 by the dot–dash lines. It can be seen from the curves that, in the second and third experiments, in which $q = \sigma \epsilon + \tau \gamma$ with a transition to the second stage becomes equal to zero, the calculated values of the deformations of the creep at the moment of change of the load instantaneously revert to zero and do not correspond to the experimental data. In the fourth and fifth experiments, with a stepwise change in the states of stress and $q > 0$, the values of the deformations calculated using (2.2)–(2.4) correspond considerably better to the experimental data than values of the deformations calculated using (1.1) and (1.2). However, Eqs. (2.2)–(2.4) do not reflect sufficiently well the experimentally observed intensification of the creep process with a change in the load.

3. In [2, 3] a theory is proposed in which the anisotropic character of the hardening of a material in the first stage of creep is taken into consideration by the introduction of an additional stress, ρ . It is postulated that the following relationships are valid:

1) the deviator of the stresses σ_{kj}^* is equal to the sum of the deviators of the active ω_{kj}^* and additional stresses ρ_{kj}^* :

$$\sigma_{kj}^* = \omega_{kj}^* + \rho_{kj}^* \quad (3.1)$$

2) the components of the deviators of the increment in the additional stress and the deformation of the creep are proportional; here, the coefficient of proportionality depends on the temperature, the intensity of the stresses, σ_i , and the intensity of the active stresses, ω_i :

$$d\rho_{kj}^* = \frac{2}{3} A(\sigma_i, \omega_i, T) d\rho_{kj} \quad (3.2)$$

3) the relationships for the components of the deformation rates of the creep have the form

$$p'_{kj} = \frac{3}{2} Q(\omega_i, T) \omega_{kj}^* / \omega_i \quad (\omega_i = (\frac{2}{3} \omega_{kj}^* \omega_{kj}^*)^{1/2}) \quad (3.3)$$

Here ω_i is the intensity of the active stresses; $Q(\omega_i, T)$ is a function of the intensity of the active stresses and the temperature.

The following expression was adopted in [3] for the function $A(\sigma_i, \omega_i, T)$:

$$A(\sigma_i, \omega_i, T) = \begin{cases} 0 & (\omega_i / \sigma_i \leq a) \\ A_1(T) A_2(\omega_i / \sigma_i) & (a < \omega_i / \sigma_i \leq 1) \\ A_1(T) & (\omega_i / \sigma_i > 1) \end{cases} \quad (3.4)$$

The function $Q(\omega_i, T)$ is sufficiently well approximated by the formula

$$Q(\omega_i, T) = C(T) \omega_i^r \quad (3.5)$$

In Eqs. (3.4) and (3.5), $a, r, C(T), A_1(T)$ are constants of the creep.

After an analysis of creep curves for AK4-1 alloy [4] at a temperature of 175°C, the following values were obtained for the constants: $a = 0.175, r = 2.6, C(T) = 2.42 \cdot 10^{-7} \text{ (mm}^2\text{r/ (kg}^r\text{· h), } A_1(T) = 4.52 \cdot 10^4 \text{ (kg/mm}^2\text{)}$.

Dependence (3.4) is shown in Fig. 4 (solid line).

With application to the experiment made, the components of the deformations of the creep were calculated using the equations

$$\varepsilon_m = \varepsilon_{m-1} + \Delta\varepsilon_m, \quad \gamma_m = \gamma_{m-1} + \Delta\gamma_m \quad (3.6)$$

$$\Delta\varepsilon_m = C(T) \{ [\sigma_m - \rho_{m-1}^{(\sigma)}]^2 + 3 [\tau_m - \rho_{m-1}^{(\tau)}]^2 \}^{(r-1)/2} [\sigma_m - \rho_{m-1}^{(\sigma)}] \Delta t_m \quad (3.7)$$

$$\Delta\gamma_m = 3C(T) \{ [\sigma_m - \rho_{m-1}^{(\sigma)}]^2 + 3 [\tau_m - \rho_{m-1}^{(\tau)}]^2 \}^{(r-1)/2} [\tau_m - \rho_{m-1}^{(\tau)}] \Delta t_m \quad (3.8)$$

$$\rho_m^{(\sigma)} = A(\sigma_i, \omega_i, T) \Delta\varepsilon_m + \rho_{m-1}^{(\sigma)}, \quad \rho_m^{(\tau)} = 1/3 A(\sigma_i, \omega_i, T) \Delta\gamma_m + \rho_{m-1}^{(\tau)} \quad (3.9)$$

where $\rho^{(\sigma)}$ and $\rho^{(\tau)}$ are the normal tangential additional stresses.

The results of a calculation using relationships (3.6)-(3.9) are shown by the dashed lines on the curves of Figs. 1, 2, and 3. As can be seen from the curves, in all the experiments, with rising stresses the agreement between the calculated and experimental data may be regarded as satisfactory. With falling stresses, there is a considerable lowering of the calculated values of the deformations with respect to the experimental.

Better agreement between the experimental and calculated data can be achieved if, for the case of falling stresses, into relationships (3.9), in place of the function $A(\sigma_i, \omega_i, T)$, we introduce the new function $B(\sigma_i, \omega_i, T)$:

$$B(\sigma_i, \omega_i, T) = \begin{cases} B_1(T) B_2(\omega_i / \sigma_i) & (0 \leq \omega_i / \sigma_i \leq 1) \\ B_1(T) & (\omega_i / \sigma_i > 1) \end{cases} \quad (3.10)$$

For AK4-1 alloy at a temperature of 175°C, the function $B(\sigma_i, \omega, T)$ is shown by the dashed line on Fig. 4. With $\omega_i / \sigma_i \geq 1, B = 10 \cdot 10^4 \text{ (kg/mm}^2\text{)}$. Calculation of the deformations of the creep with a change in the stresses from smaller to larger is done using Eqs. (3.6)-(3.9). In the case of a change in the stresses from larger to smaller, (3.9) is replaced by the relationships

$$\rho_m^{(\sigma)} = B(\sigma_i, \omega_i, T) \Delta\varepsilon_m + \rho_{m-1}^{(\sigma)}, \quad \rho_m^{(\tau)} = 1/3 B(\sigma_i, \omega_i, T) \Delta\gamma_m + \rho_{m-1}^{(\tau)} \quad (3.11)$$

The results of the calculation are represented in Figs. 2 and 3 by the solid lines a .

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